MATH 106 CALCULUS I FOR BIO. & SOC. SCI. FALL 2012

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Homework 6.

Please show all your work.

(1) Suppose that f and g are differentiable functions such that

$$f(1) = 2, f(2) = 1, f'(1) = -2, f'(2) = -1$$
$$g(1) = -2, g(2) = 2, g'(1) = 3, g'(2) = 1.$$

(a) If $a(x) = f^{-1}(x)$ compute $a'(2)$.

(b) If $b(x) = f^{-1}(\sqrt{x})$ compute $b'(4)$.

(c) If $c(x) = f \circ g^{-1}(x)$ compute $c'(2)$.

(d) If $d(x) = \ln(f(x))$ compute $d'(2)$.

(2) Consider the function $f(x) = \sqrt[3]{x}$.

(a) Find the linear approximation of f at $x = 2$.

(b) Use the above to approximate $\sqrt[3]{8}$.

(3) Find the derivative of the following functions.

(a)
$$a(x) = \ln\left(\frac{x}{x+1}\right).$$

(b)
$$b(x) = \cos(x^2)\ln(x+2).$$

(c)
$$c(x) = e^{\tan^{-1}(x)}.$$

(d)
$$d(x) = \sec^2(x\ln(x)).$$

(e)
$$e(x) = x^{2\tan(x)}.$$

(4) A scientist wants to study a certain kind of radioactive isotope present in a fossil. Let $P(t)$ denote the amount (in grams) of this isotope present in the fossil t-years after its discovery. It is known that when the rock was discovered the fossil had 2000 grams of this isotope and that the half life of this isotope is 150 years.

(a) Find an equation for $P(t)$.

(b) Find $P'(t)$.